

Hong Kong Mathematics Olympiad (2014 / 2015)

Final Event 1 (Group)

香港数学竞赛 (2014 / 2015)

决赛项目 1 (团体)

除非特别声明，答案须用数字表达，并化至最简。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 化简 $\left(\frac{1 \times 3 \times 9 + 2 \times 6 \times 18 + \dots + n \times 3n \times 9n}{1 \times 5 \times 25 + 2 \times 10 \times 50 + \dots + n \times 5n \times 25n} \right)^{\frac{1}{3}}$ 。

Simplify $\left(\frac{1 \times 3 \times 9 + 2 \times 6 \times 18 + \dots + n \times 3n \times 9n}{1 \times 5 \times 25 + 2 \times 10 \times 50 + \dots + n \times 5n \times 25n} \right)^{\frac{1}{3}}$.

2. 在 50 队香港数学竞赛的参赛队伍中，没有一队能答对一团体项目中的全部共四个题目。若该项目中的第一题有 45 队答中，第二题有 40 队答中，第三题有 35 队答中，及第四题有 30 队答中。请计算有多少队伍同时答中第三及第四题。

Among 50 school teams joining the HKMO, no one team answered all four questions correctly in the paper of a group event. If the first question was solved by 45 teams, the second by 40 teams, the third by 35 teams and the fourth by 30 teams. How many teams solved both the third and the fourth questions?

3. 设 n 为 3659893456789325678 和 342973489379256 的乘积。求 n 中数字的位数。
Let n be the product of 3659893456789325678 and 342973489379256. Determine the number of digits of n .

4. 三个半径分别为 2、3 及 10 单位的圆同时放于另一大圆内，使得四个圆都刚好彼此接触。求大圆的半径的值。

Three circles of radii 2, 3 and 10 units are placed inside another big circle in such a way that all circles are touching one another. Determine the value of the radius of the big circle.

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Final Event 2 (Group)

香港数学竞赛 (2014 / 2015)

决赛项目 2 (团体)

除非特别声明，答案须用数字表达，并化至最简。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 在一个 3×3 的方格内的九个正方形上，分别填上红色或蓝色。若 α 为不同着色方法的数量而使得所有 2×2 方格中所包含的正方形都不是全为红色，求 α 的值。

On a 3×3 grid of 9 squares, each square is to be painted with either Red or Blue. If α is total number of possible colourings in which no 2×2 grid consists of only Red squares, determine the value of α .

2. 若 25 个连续正整数之和刚好等于三个质数的积，这三个质数之和最小是多少？

If the sum of 25 consecutive positive integers is the product of 3 prime numbers, what is the minimum sum of these 3 prime numbers?

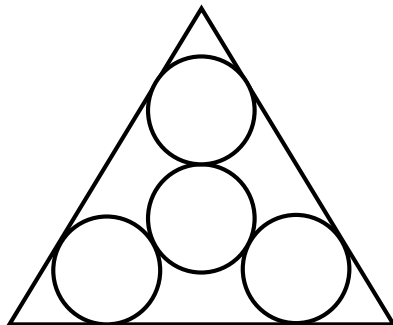
3. 求以下方程的所有实根之和：

Determine the sum of all real roots of the following equation:

$$|x+3| - |x-1| = x+1$$

4. 在下图中，四个大小相同的圆形刚好放入一个全等三角形内。若圆的半径为 1 单位，求三角形的面积的值。

In the figure below, there are 4 identical circles placed inside an equilateral triangle. If the radii of the circles are 1 unit, what is the value of the area of the triangle?



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Final Event 3 (Group)

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决赛项目 3 (团体)

除非特别声明，答案须用数字表达，并化至最简。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 化简 $\sqrt{3+\sqrt{5}} + \sqrt{3-\sqrt{5}}$ 。

Simplify $\sqrt{3+\sqrt{5}} + \sqrt{3-\sqrt{5}}$.

2. 设 p 为质 5 数及 m 为整数。若 $p(p+m)+2p=(m+2)^3$ ，找出 m 的最大可能值。

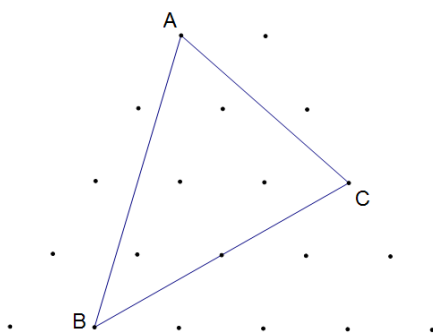
Let p be a prime and m an integer. If $p(p+m)+2p=(m+2)^3$, find the greatest possible value of m .

3. 求以下方程的根 $x = \left(x - \frac{1}{x}\right)^{\frac{1}{2}} + \left(1 - \frac{1}{x}\right)^{\frac{1}{2}}$ 。

Determine a root to $x = \left(x - \frac{1}{x}\right)^{\frac{1}{2}} + \left(1 - \frac{1}{x}\right)^{\frac{1}{2}}$.

4. 下图中，由任意彼此相邻的三点所构成的三角形的面积皆为 1。求三角形 ABC 的面积的值。

In the figure below, the area of any triangle formed by three neighbouring points is 1. Determine the value of the area of the triangle ABC .



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Final Event 4 (Group)

香港数学竞赛 (2014 / 2015)

决赛项目 4 (团体)

除非特别声明，答案须用数字表达，并化至最简。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 设 $b = 1^2 - 2^2 + 3^2 - 4^2 + 5^2 - \dots - 2012^2 + 2013^2$ 。求 b 除以 2015 的余数。

Let $b = 1^2 - 2^2 + 3^2 - 4^2 + 5^2 - \dots - 2012^2 + 2013^2$. Determine the remainder of b divided by 2015.

2. 考虑所有最大数字的数字为 6，及当把这个最大数字移除后，余下数值刚为原来数值的 $\frac{1}{25}$ 的正整数。找出在这些正整数中，数值最小的一个。

There are positive integers with leading digit being 6 and upon removing this leading digit, the resultant integer is $\frac{1}{25}$ of the original value. Determine the least of such positive integers.

3. 若 $x + \frac{1}{x} = 1$ ，求 $x^5 + \frac{1}{x^5}$ 的值。

If $x + \frac{1}{x} = 1$, determine the value of $x^5 + \frac{1}{x^5}$.

4. 在下图中，若三角形 A 向右移动 α 单位后，所形成的阴影部分的面积为三角形 A 及 B 面积总和的 α/L 。求 α/L 的值。

In the figure below, when triangle A shifts α units to the right, the area of shaded region is α/L of the total area of the triangles A and B. Determine the value of α/L .

